GET READY

Warm-Up: Distance on a Number Line

Imagine standing exactly on zero on a number line. If you must walk exactly 7 units away, where could you end up?

You could walk right and land on positive 7. You could also walk left and land on negative 7. Both numbers share the same distance from zero. This shared distance is what absolute value measures.

LEARN

Absolute Value as Distance

Imagine programming a drone that must hover exactly 10 m from a marker. It does not matter if the drone is in front of the marker or behind it. Only the distance matters.

Absolute value works the same way. The notation $|x|$ means the distance of x from zero. Because distance can go two directions, most absolute value equations have two answers.

Explicit Algebraic Breakdown

Consider the equation $2|2x-3|+5=13$. This accordion breaks down, step by step, how to solve an absolute value equation like this: first isolating the absolute value expression, then splitting it into two cases, then solving both. Click each step below to reveal how it works.

Isolate Using Reverse Order of Operations | Split Into Two Cases | Solve Both Cases

First, get the absolute value bars completely alone on one side. Use reverse order of operations: undo addition or subtraction outside the bars first, then undo multiplication or division.

For the equation $2|2x-3|+5=13$, first subtract 5 from both sides to get $2|2x-3|=8$.

Then divide both sides by 2 to get $|2x-3|=4$.

Isolate Using Reverse Order of Operations | **Split Into Two Cases** | Solve Both Cases

Once the expression is isolated, split it into two equations, one for each direction, and drop the absolute value bars.

$|2x-3|=4$ becomes $2x-3=4$ and $2x-3=-4$.

Isolate Using Reverse Order of Operations | Split Into Two Cases | **Solve Both Cases**

Solve each equation on its own.

$2x-3=4$

$2x-3=-4$

Add 3 to both sides, then divide by 2. What are the two solutions? Solve both equations to find out.

The Impossible Equation

What if isolating gives $|x-2|=-5$? Absolute value measures distance. Can distance ever be negative? Use the answer to decide whether this equation has a solution.

Real-World Tolerance

Engineers set a target size and an allowable difference called tolerance. Manufactured parts are never perfectly exact. Absolute value equations help find the acceptable range.

Consider a machine that cuts metal rods to a target length of 15 cm, with an allowable tolerance of 0.3 cm. Follow the steps to see how this scenario becomes an equation and gets solved.

Build the Equation +

Using the structure $|\text{actual} - \text{target}| = \text{tolerance}$, let x be the actual length.

The target length is 15 cm and the tolerance is 0.3 cm. Therefore, the equation is $|x - 15| = 0.3$.

Solve the Equation +

The equation $|x - 15| = 0.3$ means the actual length is 0.3 cm away from 15, in either direction. To find both possibilities, rewrite the equation as two separate equations, one for each direction, and remove the absolute value bars: $x - 15 = 0.3$ and $x - 15 = -0.3$.

Solving the first equation by adding 15 to both sides gives $x = 15.3$. Solving the second equation the same way gives $x = 14.7$. Both 15.3 and 14.7 cm are the same distance from the target, so both pass inspection.

THINK

The Three-Step Strategy

Step 1: Isolate the absolute value expression using inverse operations, in reverse order of operations.

Step 2: Split into two cases, one positive and one negative, and drop the bars.

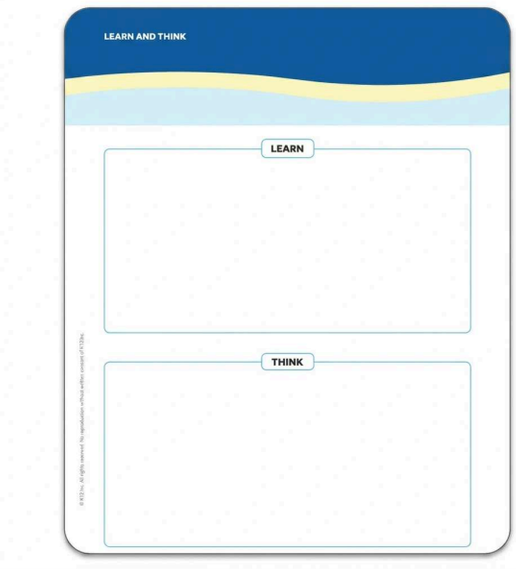
Step 3: Solve both equations to find two possible solutions.

Think about it

Why does isolating the absolute value expression have to happen before splitting into cases?

What would go wrong if you split into two cases too early, before isolating?

You can use the "Think" part of the page in the workbook to show your thinking or take notes.



Use the three-step strategy to solve $3|x+2|-4=11$. Show all work and give both values of x.

Type answer

Advanced Strategy for Writing Equations

When writing an equation from a word problem, use the structure $|\text{actual} - \text{target}| = \text{tolerance}$. The target is the ideal goal. The tolerance is the allowable room to be off.

Always pause after isolating the absolute value bars and check the sign of the number on the other side.

Think about it

If the number on the other side of the equation is negative, what does that indicate about the situation, and why?

What is a real-world tolerance scenario where zero tolerance would make sense?

Error Analysis

A student is solving $|2x + 1| + 8 = 4$.

The student subtracts 8 from both sides to get $|2x + 1| = -4$. Then the student splits the equation into $2x + 1 = -4$ and $2x + 1 = 4$.

Explain the error the student made and state the correct answer.

Type answer

TRY IT

Question 1

Solve the equation: $|x - 6| + 5 = 12$

Select one

1. $x = 1$ and $x = 11$
2. $x = 13$ and $x = -1$
3. $x = 13$ and $x = 1$
4. $x = 7$ and $x = -7$

Correct
First, subtract 5 from both sides to get $|x - 6| = 7$. Then split into $x - 6 = 7$ and $x - 6 = -7$. Adding 6 to both sides gives $x = 13$ and $x = -1$.

Incorrect
Start by subtracting 5 from both sides to isolate the absolute value bars. Once the bars are alone, set the inside expression equal to both the positive and negative results before solving for x.

Question 2

Solve the equation: $2|3x| + 14 = 8$

Select one

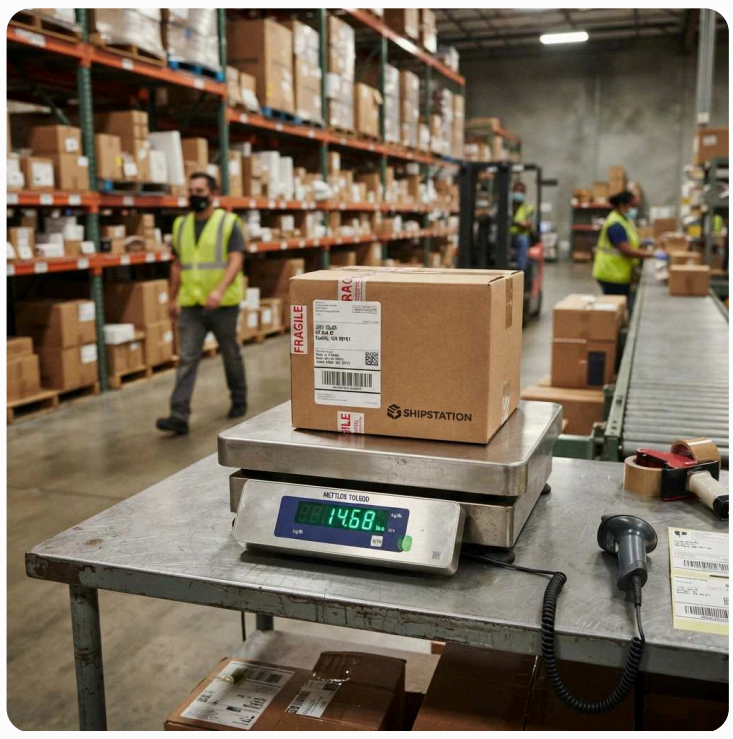
1. $x = 1$ and $x = -1$
2. $x = 3$ and $x = -3$
3. $x = -1$
4. No solution

Correct
Subtracting 14 from both sides gives $2|3x| = -6$. Dividing by 2 gives $|3x| = -3$. Because an absolute value cannot equal a negative number, there is no solution.

Incorrect
Try isolating the absolute value expression first using reverse order of operations. Subtract 14, then divide by 2. Examine the number on the right side of the equal sign. Can distance ever be negative?

Question 3

A shipping company requires packages to weigh 25 pounds, with an allowable variance of 2 pounds. Write an absolute value equation that models this situation, then solve it to find the minimum and maximum acceptable weights. In one sentence, explain why the tolerance value is what determines how far the weight can be from the target.



Type answer

Question 4

The shipping company decides to double the allowable variance from Question 3, while keeping the target weight at 25 pounds. Write a new absolute value equation that models this updated situation.

Type answer

Wrap-Up

Remember the three-step strategy: isolate, split, solve. Always check the sign after isolating. If the isolated absolute value expression equals a negative number, the equation has no solution.